

**EXAMPLE ENTRY QUALIFICATION TEST**  
**Second Cycle Studies, Data Science**

1. *This task is about object programming skills. Please design your solution in clean, extendable way so little or no code is repeated.*

Propose a class hierarchy to implement typical arithmetic operators: addition, subtraction, multiplication, division, factorial. These operators should be nodes in a tree of an equation. So, e.g., addition node has two children: numbers or further operators. There should be a method `calculate_value()` for each operator, as well as `print()`, which prints the name of the operator and its children.

Example: Equation "2+4\*6!" will be stored in such a tree (root is +, you should read it as `node(child1, child2)`), i.e., `+(2, *(4, !(6)))`. Note that, the `calculate_value()` should return 2882.

2. *This task is about implementing a simple genetic algorithm for travelling salesman problem. This is a problem where  $n$  cities and all distances between them are given. We would like to start from one of these cities and visit every one of them exactly once (and return to the starting city). Of course to obtain exact solution  $n!$  possible solutions should be tested, which is impossible for greater values of  $n$ .*

Your task is to propose a simple genetic algorithm for this problem: we start with  $m$  randomly chosen solutions (we call them "population") and in every iteration (let us say there is  $I$  iterations) we do as follows:

- (a) Choose randomly 2 solutions and crossover them. It means: use some part of the first solution and some part of the second solution and combine them into a one, new solution.
- (b) Mutate the solution obtained in the previous step. It means: make a simple alteration in this solution, so it is some different, proper solution.
- (c) From three solutions (two parents and one mutated child) choose two best solutions and leave them in population, so its count does not change.

After  $I$  iterations return the best solution from the population.

This is your job to propose how to store solutions, how to store or calculate distances between cities and propose crossover and mutation operator.

3. The probability of a small town bank being robbed is 0.3. In order to protect the bank from theft, a modern alarm system was installed. Unfortunately, however, the system has some drawbacks, i.e., the system will fail in 2% cases when there is an attempted robbery. On the other hand, when there is no threat of theft, the system will unnecessarily activate the alarm with probability 0,05. Calculate the probability that the alarm sounded by the system turns out to be false.

4. It is estimated that 40% of taxpayers will receive a refund for overpaid taxes. Among 800 randomly selected taxpayers, what is the probability that more than 300 but not more than 400 people are due a refund?
5. For which values of the parameter  $p$  the system of equations

$$\begin{cases} px + y + z = 1 \\ x + py + z = p \\ x + y + pz = p^2 \end{cases}$$

has no solutions, exactly one solution, infinitely many solutions? Find the subspace of solutions in the case there are infinitely many solutions.

6. Calculate the integral

$$\int \ln(x^2 + 2x + 10) dx.$$