Report on scientific activities of Dr. hab. Ewa Bednarczuk

Following her application for the Professor Degree in Poland

Dr. hab Ewa Bednarczuk has completed her habilitation in 2006 at System Research Institute of Polish Academy of Sciences. Since then she has published 19 papers and one monograph. Three more manuscripts were submitted in 2018 and are still under the evaluation process in journals. All, but two, of them are written in collaboration either with her PhD students or senior scientists (A. Kruger, A. Tretyakov, D. Zagrodny, to cite a few). Most of these papers were published in high level international journals of applied mathematics SIOPT, SICON, NA TMA, Convex Analysis or in Special issues of good level. All together, since 1977, E. Bednarczuk has written 47 papers and two monographs.

1. Scientific achievements

The scientific research area of E. Bednarczuk is the abstract optimization and, in particular, the vector optimization in infinite dimensional framework. In her research she applies methods of modern variational analysis in the context of mappings with values in vector spaces. This naturally leads to various generalizations of the celebrated Ekeland variational principle and also of the Borwein-Preiss principle, vector-Palais-Smale conditions and vector versions of the mountain-pass lemma, questions of differentiability of "cone-convex" mappings, slops of the vector valued mappings useful in error estimates, etc. She is an internationally recognized specialist of vector optimization and is often invited to give talks at international conferences.

Below just some of these topics are discussed in more detail to describe some of the scientific achievements of E. Bednarczuk.

The Ekeland variational principle (1974) is an elegant tool used in many applications, and, in particular, in partial differential equations, optimization, and control theory. It involves a penalty term expressed via the distance function, which is not differentiable. In 1987 it was proved by Borwein and Preiss that a somewhat similar result can be obtained with a smoother penalty term. However this time one has to use series and applications of this variational principle mostly concerned the subdifferential calculus. A number of researcher generalized the Ekeland principle to the case of vector spaces, were the "natural" order on scalars was replaced by an order induced by convex cones.

E. Bednarczuk together with co-authors obtained several extensions of both principles to normed vector spaces. In particular, when the order is given by a pointed convex cone with

nonempty interior, in the obtained extension of the Ekeland principle the minimizer of the penalized function is proved to be sharply efficient, which strengthen substantially the earlier results. For the case of convex topological spaces the obtained extension concerns perturbation in the directions of some bounded subsets of the ordering cone, instead of one single direction like in the existing literature. This allows a much larger spectrum of applications.

Also a version of the Borwein-Preiss principle was obtained by E. Bednarczuk (in collaboration with D. Zagrodny) for reflexive Banach spaces ordered by a convex normal cone. This version is more precise and far more general than the original one. No applications were provided so far but one can anticipate that it could be useful for building the subdifferential calculus in vector spaces.

Another direction of research of E. Bednarczuk concerns optimization with degenerate constraints. This is based on p-regularity theory developed by Tretyakov since 1983. Together with A. Tretyakov (and, sometimes, with Prusinska) she investigated mathematical programming problems involving equality constraints with degenerate derivatives. One should mention here that the classical result by Lusternik expresses tangents as the kernel of the first derivative. However this result requires a surjectivity assumption, that is non-degeneracy. Thanks to this new approach the first order necessary and the second order sufficient conditions for optimality were derived involving higher order derivatives of degenerate constraints. When the sufficient condition is verified these results implied the Holder continuity of local minimizers with respect to a perturbation parameter. Such stability results may be useful in numerical approximation of minimizers. Some elements of these techniques were also applied to the problem of reduction of degenerate problems to regular ones and also to an implicit function theorem in the degenerate case.

Also, on the basis of p-regularity theory, a subset of tangents to sets described by equalities was expressed in an elegant way in the presence of degeneracies.

Same approach was applied then to particular classes of constrained calculus of variation problems that are p-regular. These results in continuous optimization are different from the usual Euler-Lagrange equations in the presence of non-degenerate state constraints, where one deals with measures, see for instance Vinter's book on optimal control theory. Furthermore, second order conditions are also usually obtained in the non-degenerate case by imposing constraint qualifications of the Mangasarian Fromovitz type. The results derived by E. Bednarczuk allow some degeneracy of data. At the same time they are not yet general enough to be convinced that this investigation solved the difficulties due to degeneracy of constraints. For instance the existence theorems in the Calculus of Variations do not guarantee continuous differentiability of optimal solutions. However this framework is imposed in main results and so further study is needed here. Also several other assumptions replaced the non-degeneracy requirements.

In collaboration with K. Leśniewski, E. Bednarczuk obtained results on the Gateaux differentiability on G-delta sets of strongly cone-paraconvex functions (for convex normal ordering cone) taking values in a separable reflexive Banach space. Furthermore, if the ordering cone has a bounded base and the mapping is defined on a separable Asplund space, then a similar result is valid aslo for the Fréchet derivative. These are nontrivial extensions of the earlier results by S. Rolewicz obtained for scalar valued (or finite dimensional space valued mappings). Definition of cone paraconvex functions in the context of vector spaces is very similar to the one of semiconvex functions. Taking the opposite sign, a semiconvex function becomes semiconcave. Semiconcave functions, in turn, are very useful in investigation of some first order PDEs related to optimal control problems. For instance it is

known, cf. L. Evans, that for Hamilton-Jacobi-Bellman equations semiconcave solutions are uniquely defined by the semiconcave initial conditions. E. Bednarczuk has shown that strongly cone paraconvex functions do have directional derivatives at every point. Further, when the ordering cone is closed normal and convex, any cone-paraconvex function taking values in a weakly sequentially complete Banach space has directional derivatives. Finally, weakly sequentially complete Banach spaces Y were characterized using the existence of directional derivatives of K-convex mappings from X to Y for any closed convex cone K in Y. The reviewer of this last paper in Mathematical Reviews is quite critical about the assumptions. It could be interesting to derive next an analogue of Hamilton-Jacobi-Bellman equation in the context of vector spaces, since lots of knowledge is already accumulated here and multicriteria problems do often arise in the industrial applications.

In the last thirty years part of the international scientific community got actively involved in the research on metric regularity. In this area E. Bednarczuk has worked on so called error bounds in collaboration with A. Kruger, the topic important for investigation of convergence of numerical methods, stability and subdifferential calculus. Here she extended the notion of slop to vector valued mappings and got the estimates of the error bounds using these slops.

The above highlights do indicate that E. Bednarczuk has investigated a large spectrum of problems in the context of vector spaces and obtained various generalizations and improvements using modern methods of variational analysis. She also knew to attract PhD students to this area of research.

According to Mathematical Reviews E. Bednarczuk was cited 279 times by 188 authors. Her most cited papers are those written in collaboration with Penot, Song and Zagrodny.

2. Supervision of students

Dr. hab Ewa Bednarczuk has supervised two doctoral dissertations defended in 2016 and 2018 and is presently supervising two more doctoral students. In 2018 the thesis of K. Leśniewski got a distinction from the faculty MiNI PW.

She was also involved as external reviewer in three PhD thesis committees and one habilitation committee in Vietnam. Beside that she was also asked to review research activities of an Italian mathematician applying for the professorship.

3. Final conclusions.

In this referee opinion Dr. hab Ewa Bednarczuk deserves the title of Professor on the basis of her scientific achievements and her care about the young researchers.

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